Constructive Computer Architecture

Combinational ALU

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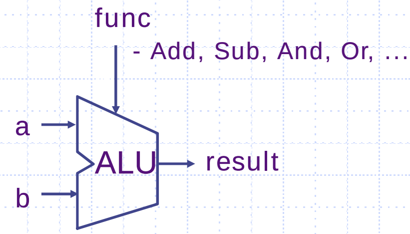
Outline

- Building complex combinational circuits in pieces
- Parameterization that goes beyond data path widths

Arithmetic-Logic Unit (ALU)

ALU performs all the arithmetic and logical functions

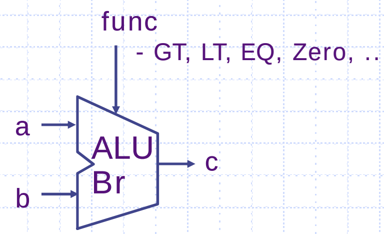



```
function Data alu(Data a, Data b,
                  AluFunc func);
```

Each individual function can be described as a combinational circuit and these can be combined together to produce a combinational ALU

ALU for comparison operators

Like ALU but returns a Bool




```
function Bool aluBr(Data a, Data b,
                    BrFunc func);
```

what does func look like?

Enumerated types

- Suppose we have a variable c whose values can represent three different colors
  - Declare the type of c to be Bit#(2) and adopt the convention that 00 represents Red, 01 Blue and 10 Green
- A better way is to create a new type called Color:

```
typedef enum {Red, Blue, Green}
Color deriving(Bits, Eq);
```

BSV compiler automatically assigns a bit representation to the three colors and provides a function to test if the two colors are equal

If you do not use "deriving" then you will have to specify your own encoding and equality function

Enumerated types

```
typedef enum {Red, Blue, Green}
Color deriving(Bits, Eq);

typedef enum {Add, Sub, And, Or, Xor, Nor, Slt, Sltu,
LShift, RShift, Sra} AluFunc deriving(Bits, Eq);

typedef enum {Eq, Neq, Le, Lt, Ge, Gt, AT, NT}
BrFunc deriving(Bits, Eq);
```

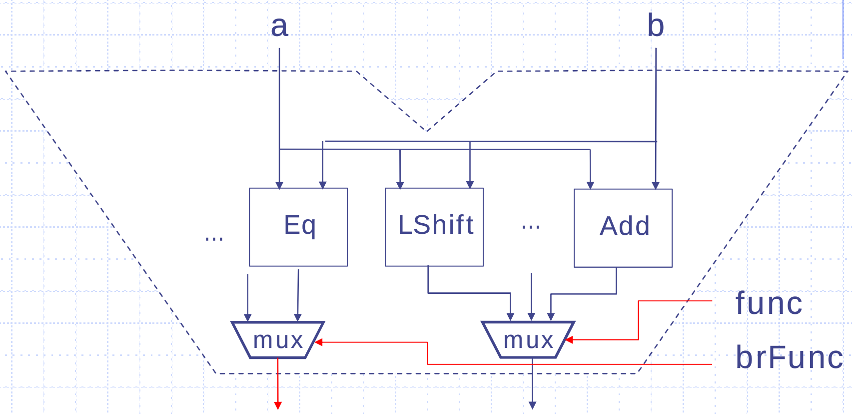
Combinational ALU

```
function Data alu(Data a, Data b, AluFunc func);
  Data res = case(func)
     Add    : addN(a,b);
     Sub    : subN(a,b);
     And    : andN(a,b);
     Or     : orN(a,b);
     Xor    : xorN(a,b);
     Nor    : norN(a,b);
     Slt    : zeroExtend(pack(signedLT(a,b)));
     Sltu   : zeroExtend(pack(lt(a,b));
     LShift: shiftLeft(a,b[4:0]);
     RShift: shiftRight(a,b[4:0]);
     Sra    : signedShiftRight(a,b[4:0]);
  endcase;
  return res;
endfunction
```


Comparison operators

```
function Bool aluBr(Data a, Data b, BrFunc brFunc);
    Bool brTaken = case(brFunc)
      Eq  : (a == b);
      Neq : (a != b);
      Le  : signedLE(a,b);
      Lt  : signedLT(a,b);
      Ge  : signedGE(a,b);
      Gt  : signedGT(a,b);
      AT  : True;
      NT  : False;
    endcase;
    return brTaken;
  endfunction
```

ALU including Comparison operators

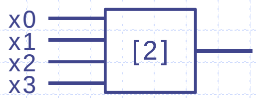


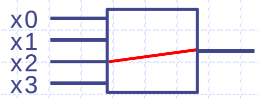
Selectors and Multiplexers

Selecting a wire: x[i]

assume x is 4 bits wide

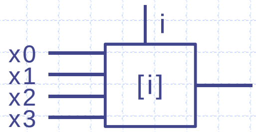
- Constant Selector: e.g., x[2]

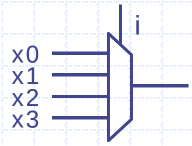




no hardware; x[2] is just the name of a wire

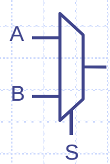
- Dynamic selector: x[i]

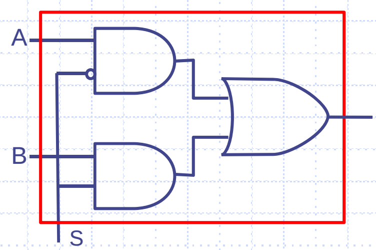




4-way mux

A 2-way multiplexer



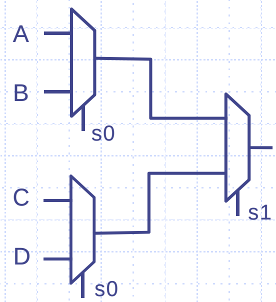


(s==0)? A : B ;

Gate-level implementation

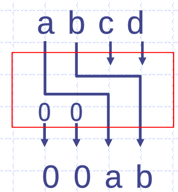
A 4-way multiplexer

```
case ({s1,s0}) matches
    0:  A;
    1:  B;
    2:  C;
    3:  D;
endcase
```



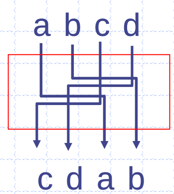
Shift operators

Logical right shift by 2

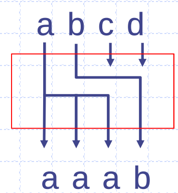


- Fixed size shift operation is cheap in hardware – just wire the circuit appropriately
- Other types of shifts are similar

Rotate



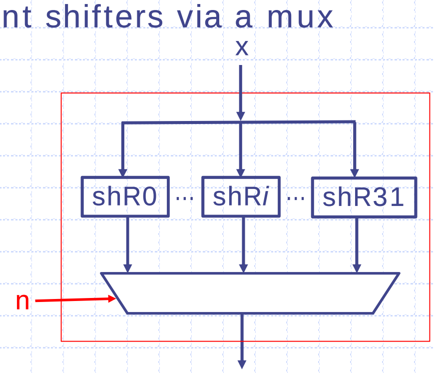
Sign-extended



Sign extension is useful for converting say, a 32-bit integer into a 64-bit integer

Logical right shift by *n*

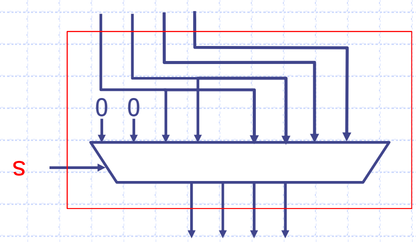
- Suppose we want to build a shifter which shift a value x by *n* where n is between 0 and 31
- One way to do this is by connecting 31 different shifters via a mux



Logical right shift by *n*

- Shift *n* can be broken down in log *n* steps of fixed-length shifts of size 1, 2, 4, …
  - For example, we can perform Shift 3 (=2+1) by doing shifts of size 2 and 1
  - Shift 5 (=4+1) by doing shifts of size
  - Shift 21 (=16+4+1) by doings shifts of size
- For a 32-bit number, a 5-bit *n* can specify all the needed shifts
  - $3_{10} = 00011_2$, $5_{10} = 00101_2$, $21_{10} = 10101_2$
- The bit encoding of *n* tells us which shifters are needed; if the value of the i^{th} (least significant) bit is 1 then we need to shift by 2^i bits

Conditional operation: shift versus no-shift

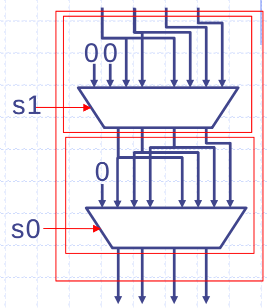


- We need a mux to select the appropriate wires: if s is one the mux will select the wires on the left otherwise it would select wires on the right

```
(s==0)?{a,b,c,d}:{0,0,a,b};
```

Logical right shift ckt

- Define log *n* shifters of sizes 1, 2, 4, …
- Define log *n* muxes to perform a particular size shift
- Shift circuit can be expressed as log *n* nested conditional expressions where s0, s1 .. Represent the bits of *n*



Complex Combinational Circuits

Multiplication by repeated addition

```
b Multiplicand      1101   (13)
a Muliplier     *   1011   (11)

                    0000
                x   1101
    ------------------------
 m0                 1101
 m1             +  1101
 m2             + 0000
 m3             + 1101
    ------------------------
                10001111   (143)
```

At each step we add either 1101 or 0 to the result depending upon a bit in the multiplier

```
mi = (a[i]==0)? 0 : b;
```

We also shift the result by one position at every step

However, our addN circuit adds only two numbers at a time!

Multiplication by repeated addition *cont.*

```
b Multiplicand      1101   (13)
a Muliplier     *   1011   (11)

tp                  0000
m0              +   1101
    ------------------------
tp                 01101
m1              +  1101
    ------------------------
tp
m2
    ------------------------
tp
m3
    ------------------------
tp
```

At each step we add either 1101 or 0 to the result depending upon a bit in the multiplier

```
mi = (a[i]==0)? 0 : b;
```

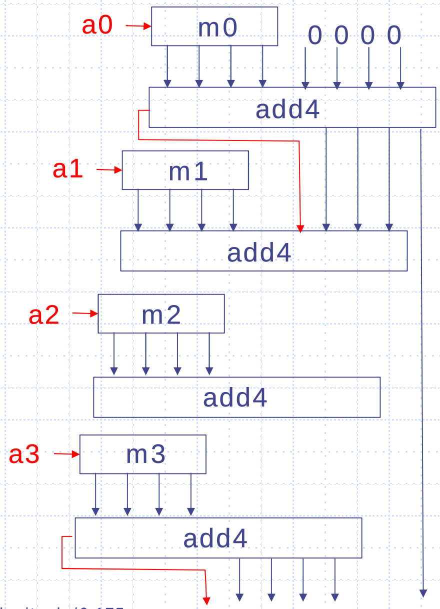
We also shift the result by one position at every step

Multiplication by repeated addition ckt

```
b Multiplicand    1101   (13)
a Muliplier   *   1011   (11)

tp                0000
m0        +       1101
tp               01101
m1        +     1101
tp              100111
m2        +   0000
tp             0100111
m3        +  1101
tp            10001111   (143)
```

$\boxed{\texttt{mi = (a[i]==0)? 0 : b;}}$



Combinational 32-bit multiply

```
function Bit#(64) mul32(Bit#(32) a, Bit#(32) b);
  Bit#(32) tp = 0;
  Bit#(32) prod = 0;
  for(Integer i = 0; i < 32; i = i+1)
  begin
     Bit#(32) m   = (a[i]==0)? 0 : b;
     Bit#(33) sum = add32(m,tp,0);
     prod[i]      = sum[0];
     tp           = sum[32:1];
  end
  return {tp,prod};
endfunction
```

- Long chains of gates
  - 32-bit multiply has 32 ripple carry adders in sequence!
  - 32-bit ripple carry adder has a 32-long chain of gates
  - Total delay ?